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Solute dispersion along unsteady groundwater flow in a semi-infinite aquifer

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Abstract

Analytical and numerical solutions are obtained for dispersion of pollutants along unsteady groundwater flow in a longitudinal direction through semi-infinite aquifers the permeability of which is either uniform or varies with position. Sources of pollution are both a concentrated point input at the origin and a spatially distributed background source. One expression chosen to represent the seasonal pattern of the time dependent velocity is sinusoidal behaviour over a year. The solutions obtained predict the time and distance from the location at which an input concentration is introduced at which the pollution concentration becomes harmless. Also, the time period for rehabilitating a polluted aquifer for human use can also be assessed.

Introduction

Allthough in some regions groundwater varies with time, relatively few solutions for solute dispersion in aquifers in such regions have been reported. In tropical regions such as India, groundwater flow and level show a seasonal variation of sinusoidal nature around a year. Groundwater level and velocity are at maximum in the middle of winter, after the rainy season and the two are minimum in summer, just before the rainy season. The ultimate source of water to sustain a groundwater body in fine to coarse grained sands of the older alluvium is rainfall and infiltration from rivers in spate. The Gangetic Basin, a part of Himalayan foredeep which covers an area of about 2,50,000 km^2 between the Himalayan front and the Indian peninsular shield, is one of the longest groundwater reservoirs in the world. Even in recent publications like Leij *et al.* (1993), Aral and Tang (1993) and Serrano (1995) the groundwater velocity is considered steady. Van Genucheten and Alves (1982) and Javendel *et al.* (1984) have reviewed and compiled most of the non-dimensional conventional dispersion problems for steady flow through porous media. Fry *et al.* (1993) presented model equations and their analytical solutions; the solutions depend on the magnitudes of the model parameters. On the other hand, experimentally, Banks and Jerasate (1962) derived linear and exponentially decreasing time dependent expressions for seepage velocity through porous media. Oroveanu (1966) has shown how seepage varies inversely as the square of time. Bear (1972) proposed that, in some conditions, permeability, hence the seepage velocity through a porous medium, may vary with time due to external loads that produce stresses that change the texture and structure of the porous medium.

The present work deals with one-dimensional solute dispersion along unsteady groundwater flow in a semi-infinite and saturated aquifer of substantial depth. Sinusoidal and exponentially decreasing time-dependent expressions for groundwater velocity are considered separately. The former represents the seasonal pattern of sinusoidal fluctuations in groundwater velocity over a year. Analytical solutions are provided for a homogeneous aquifer, while for an inhomogeneous one, numerical solutions are obtained. The semi-infinite system is subject to zero-order production. A direct relationship between dispersion coefficient and velocity helps to convert the time dependent coefficients in the governing equations into constant coefficients. Such a relationship has been established by Ebach and White (1958), Scheidegger (1961), Rumer (1962) and Bruce (1970). Rumer (1962) found that such a relationship for steady flow was also valid for unsteady flow with sinusoidally or exponentially varying velocity through porous media.

Mathematical formulation

Pollutants from instantaneous point sources (spills), such as septic tanks, garbage disposal sites, cemeteries and mine spoils on the surface; infiltrate to groundwater and spread along the flow. The solute concentration distribution due to dispersion and convection in one space dimension, along with a zero-order liquid phase source, is defined by a partial differential equation of parabolic type (Scheidegger, 1961; Bachmat and Bear, 1964).

$$\frac{\partial c^*}{\partial t} = \frac{\partial}{\partial x}\left[D\frac{\partial c^*}{\partial x} - uc^*\right] + \gamma^* \quad (1)$$

where $c^*(ML^{-3})$ is solute concentration in the liquid phase, $\gamma^*(ML^{-3}T^{-1})$ is a zero-order production term, $D(L^2T^{-1})$ is the dispersion coefficient and $u(LT^{-1})$ is the groundwater velocity at position x and time t. Initially, groundwater is not solute free due to some internal cause or effect in the aquifer, or some other type of zero-order production, represented by the symbol γ^*. This produces solute particles in the liquid phase. An appropriate initial condition may be chosen as

$$c^*(x, 0) = C_1 + \gamma^* x/u;\ x \geq 0 \quad (2)$$

The input concentration at the origin (where the pollutants reach the groundwater level) is of pulse type. The first boundary condition is defined as at

$$x = 0,\quad -(1-\delta)(D/u)\frac{\partial c^*}{\partial x} + c^* = C_0 f(t);\quad 0 < t \leq t_0 \quad (3a)$$

$$= 0; \qquad t > t_0 \quad (3b)$$

t_0 is the time at which the source of the pollution on the surface is eliminated for ever. For $\delta = 1$ the conditon (3a) is of the first or function-type where the solution is prescribed at the origin. When the derivative of the solution is defined at the boundary, it is of the second or flux type boundary condition. A third or mixed-type boundary condition is that in which a linear combination of solution and its derivative is defined at the boundary. The condition (3a) is of mixed-type for $\delta = 0$. When the conditions (3) are of the first-type, it means that the input concentration at the groundwater level and hence its source of pollution on the surface remain uniform during $0 < t \leq t_0$, and the input becomes zero immediately after the source is eliminated ($t > t_0$). But this may not be the real situation. In fact, with increasing human activities on the surface, pollution at the source and so the input at the groundwater level will increase during $0 < t \leq t_0$ and the input, instead of becoming zero, will start decreasing when $t > t_0$. The infiltrated concentration reaching the groundwater level penetrates downward because of higher density and from each point of the downward vertical, starts to spread longitudinally, along the flow in the region $x > 0$. Naturally some concentration spreads within the nearby region $x < 0$. As soon as the infiltration starts decreasing and eventually stops due to elimination of the pollutant source on the surface (at $t = t_0$), this concentration in the $x < 0$ region starts crossing the boundary $x = 0$ and spreads along the flow. That is why, after $t = t_0$, up to some duration, the input concentration remains at $x = 0$, though it decreases with time. Only after a long time period may it become zero. This realistic picture can be demonstrated by the mixed type boundary conditions (3). Such conditions help to predict the time period under which a polluted aquifer can be rehabilitated.

The other boundary condition for a semi-infinite system that is subject to zero-order production can be defined as

$$\frac{\partial c^*}{\partial x} = \text{finite} \quad \text{as } x \to \infty; \quad t \geq 0 \quad (4)$$

The two forms of unsteady groundwater velocity are considered as

$$u(t) = u_0\,(1 - \sin mt) \quad (5a)$$

$$u(t) = u_0 \exp(-mt),\quad mt < 1, \quad (5b)$$

where u_0 is the initial velocity and $m(T^{-1})$ is the flow resistance coefficient.

Dispersion in homogeneous aquifer

The partial differential equation (1) when D and u are independent of x, for an homogeneous aquifer can be written as:

$$\frac{\partial c^*}{\partial t} = D\frac{\partial^2 c^*}{\partial x^2} - u\frac{\partial c^*}{\partial x} + \gamma^* \quad (6)$$

where
$$u = u_0\, V(t) \quad (7)$$

As dispersion coefficient varies directly with velocity, let $D = \alpha u$, α is a coefficient of the dimension of length and depends upon pore-system geometry and on the average pore-size diameter of an aquifer. Using the expression (7). $D = D_0 V(t)$, where $D_0 = \alpha u_0$ is the initial dispersion coefficient and $\gamma^* = \gamma_0 V(t)$, when γ_0 is the initial zero order production term, Eqn. (6) becomes,

$$\frac{1}{V(t)}\frac{\partial c^*}{\partial t} = D_0\frac{\partial^2 c^*}{\partial x^2} - u_0\frac{\partial c^*}{\partial x} + \gamma_0 \quad (8)$$

Introducing a new time variable by the following transformation (Crank, 1975)

$$T^* = \int_0^t V(t)dt \quad (9)$$

The equation (8) assumes the form

$$\frac{\partial c^*}{\partial T^*} = D_0\frac{\partial^2 c^*}{\partial x^2} - u_0\frac{\partial c^*}{\partial x} + \gamma_0 \quad (10)$$

Now, the above equations are non-dimensional but not in the same way as in the Leij *et al.* (1993) work on semi-infinite or infinite domains, where space and time variable are non-dimensionalised in terms of parameter L (length of aquifer) which does not occur in the problem. This flaw is taken care of in the present problem and non-dimensional variables are introduced in terms of existing parameters as follows:

$$C = \frac{c^*}{C_0},\quad X = \frac{xu_0}{D_0},\quad T = \frac{u_0^2 T^*}{D_0};\quad \gamma = \frac{\gamma_0 D_0}{C_0 u_0^2} \tag{11}$$

In terms of these, Eqn. (10) can be written as

$$\frac{\partial C}{\partial T} = \frac{\partial^2 C}{\partial X^2} - \frac{\partial C}{\partial X} + \gamma \tag{12}$$

and the initial and boundary conditions (2–4) can be deduced as

$$\text{at } T = 0,\ C(X, T) = C_1/C_0 + \gamma X \quad ; X \geq 0 \tag{13}$$

$$\text{at } X = 0,\ -(1-\delta)(\partial C/\partial X) + C = f(T);\ 0 < T \leq T_0 \tag{14a}$$

$$= 0 \quad ; T > T_0 \tag{14b}$$

$$\text{and as } X \to \infty;\ \partial C/\partial X = \text{finite} \quad ; T \geq 0 \tag{15}$$

Now, a Laplace transformation is used to obtain the analytical solutions of (12–15) for different cases.

Case 1: $V(t) = 1 - \sin mt$, i.e. a sinusoidal form (5a) for groundwater velocity is considered. The non-dimensional time variable defined in (11) can be obtained as

$$T = (u_0^2 / D_0)[t - (1 - \cos mt)/m] \tag{16}$$

Also $f(t)$ in (3a) is taken as unity, i.e. in (14a) $f(T) = 1$.

The analytical solution for $\delta = 0$ in (14a) i.e. for mixed type boundary conditions can be obtained as

$$C(X,T) = \frac{C_1}{C_0} + \gamma X + \left(1 + \gamma - \frac{C_1}{C_0}\right) F(X,T);$$
$$0 < T \leq T_0 \tag{17a}$$
$$= \frac{C_1}{C_0} + \gamma X + \left(1 + \gamma - \frac{C_1}{C_0}\right)$$
$$F(X,T) - F(X, T - T_0);\quad T > T_0 \tag{17b}$$

where

$$F(X,T) = \sqrt{T/\pi}.\exp\left(\frac{-(X-T)^2}{4T}\right) + E_1 - \exp(X) E_2,$$

E_1 stands for

$$\tfrac{1}{2}\operatorname{erfc}\left(\frac{X}{2\sqrt{T}} - \frac{\sqrt{T}}{2}\right)$$

and E_2 for

$$\tfrac{1}{2}\operatorname{erfc}\left(\frac{X}{2\sqrt{T}} + \frac{\sqrt{T}}{2}\right)$$

in the above and subsequent solutions.

For $\delta = 1$ (concentration type condition) in (14a), the analytical solutions will be

$$C(X,T) = \frac{C_1}{C_0} + \gamma X + \left(1 - \frac{C_1}{C_0}\right) F(X,T);\quad 0 < T \leq T_0 \tag{18a}$$
$$= \frac{C_1}{C_0} + \gamma X + \left(1 - \frac{C_1}{C_0}\right) F(X,T) - F(X, T - T_0);$$
$$T > T_0 \tag{18b}$$

where $F(X, T) = E_1 + \exp(X) E_2$.

Case 2: $V(t) = \exp(-mt)$, $mt < 1$ i.e. groundwater velocity has the form (5b). The function f(t) in the boundary condition (3a) is chosen as $[1 - \exp(-qt)]$, $qt < 1$. From the transformation (9),

$$T^* = \int_0^t \exp(-mt)dt = \frac{1}{m}[1 - \exp(-mt)] \tag{19}$$

So $\quad f(t) = 1 - \exp(-qt) = 1 - (1 - mT^*)^{q/m}$

In this case, both parameters m and q will be either of the same order or equal. Let $m = 0.0002$ (days)$^{-1}$ and $q = 0.0001$ (days)$^{-1}$. For $t = 1000$ days (let), mt and qt, both are much less than one. From eqn. (19), $qT^* = 0.0906$ and $\frac{1}{2}q(m - q)T^{*2} = 0.0041$ which is much less than qT^*. So the higher order terms in the binomial expression $(1 - mT^*)^{q/m}$ can be neglected as compared to qT^*. When $m = q$, all the terms containng $(m - q)$ will become zero. Thus, $f(t) = qT^*$, and the boundary condition (14a) contains the expression $f(T) = QT$ where the non-dimensional parameter $Q = qD_0/u_0^2$. For $\delta = 0$ (i.e. for the mixed type boundary condition) the analytical solutions will be

$$C(X,T) = \frac{C_1}{C_0} + \gamma X + \left(\gamma - \frac{C_1}{C_0}\right) F(X,T) + QG(X,T); \tag{20a}$$
$$0 < T \leq T_0$$
$$= \frac{C_1}{C_0} + \gamma X + \left(\gamma - \frac{C_1}{C_0}\right) F(X,T)$$
$$-QT_0 F(X, T - T_0) + Q[G(X,T)$$
$$-G(X, T - T_0)];\quad T > T_0 \tag{20b}$$

where

$$F(X,T) = \sqrt{T/\pi}.\exp\left(\frac{-(X-T)^2}{4T}\right)$$
$$+E_1 - (1 + X + T)\exp(X) E_2,$$
$$G(X,T) = (1 + X/2 + T/2)\sqrt{T/\pi}\exp\left(\frac{-(X-T)^2}{4T}\right)$$
$$+(T - X - 1)E_1$$
$$-[T - 1 + \tfrac{1}{2}(X+T)^2]\exp(X) E_2$$

Similarly the analytical solutions when $\delta = 1$, (i.e. for the concentration type boundary condition) exist as

$$C(X,T) = \frac{C_1}{C_0} + \gamma X - \frac{C_1}{C_0} F(X,T) + QG(X,T);$$
$$0 < T \leq T_0 \tag{21a}$$

$$= \frac{C_1}{C_0} + \gamma X - \frac{C_1}{C_0} F(X,T) - QT_0 F(X, T - T_0)$$
$$+Q[G(X,T) - G(X, T - T_0)];\quad T > T_0 \tag{21b}$$

where $F(X, T) = E_1 + \exp(X)E_2$.
and $G(X, T) = (T - X)E_1 + (T + X)\exp(X)E_2$.

The non-dimensional time variable in the solutions of case 2, has the expression,

$$T = (u_0^2 / mD_0)[1 - \exp(-mt)] \qquad (22)$$

These solutions match those for steady groundwater flow compiled by Van Genucheten and Alves (1982), if $m = 0$ is substituted in the expressions for velocity (5a, b).

Dispersion along inhomogeneous aquifer

In an homogeneous aquifer in which permeability varies with position, the dispersion coefficient as well as groundwater velocity will be functions of x, both having the same tendency. Let the two be defined as

$$D = D(t)F(x) \text{ and } u = u(t)F(x) \qquad (23)$$

The partial differential equation (1) will be written as

$$\frac{\partial c^*}{\partial t} = F(x)\left[D\frac{\partial^2 c^*}{\partial x^2} - u\frac{\partial c^*}{\partial x}\right] + \frac{d}{dx}F(x)\left[D\frac{\partial c^*}{\partial x} - uc^*\right] + \gamma^* \qquad (24)$$

which, in terms of non-dimensional variables defined in eqn. (11), can be written as

$$\frac{\partial C}{\partial T} = F(X)\left[\frac{\partial^2 C}{\partial X^2} - \frac{\partial C}{\partial X}\right] + \frac{d}{dX}F(X)\left[\frac{\partial C}{\partial X} - C\right] + \gamma \qquad (25)$$

Following two expressions of $F(X)$ are defined

$$F(x) = 1 - \frac{0.5\exp(-X)}{1.5 + \exp(-X)} \qquad (26a)$$

$$F(x) = 0.8 - \frac{0.05\exp(-X)}{1.25 - \exp(-X)} \qquad (26b)$$

The former expression increases from 0.8 at $X = 0$ to 1.0 as $X \to \infty$ while the latter expression has the reverse tendency. For numerical solutions, a finite difference scheme (two-level explicit) is used. But the semi-infinite domain $X \in (0, \infty)$ is converted into a finite domain $Y \in (0, 1)$ by the following transformation

$$Y = 1 - \exp(-X) \qquad (27)$$

The partial differential equation (25) converts as follows

$$\frac{\partial C}{\partial T} = (1-Y)\left[F(Y)\left\{(1-Y)\frac{\partial^2 C}{\partial Y^2} - 2\frac{\partial C}{\partial Y}\right\}\right] + \frac{d}{dY}F(Y)\left[(1-Y)\frac{\partial C}{\partial Y} - C\right] + \gamma \qquad (28)$$

where

$$F(Y) = 1 - \frac{0.5(1-Y)}{2.5 - Y} \qquad (29a)$$

and

$$F(Y) = 0.8 + \frac{0.05(1-Y)}{0.25 - Y} \qquad (29b)$$

$F(Y)$ has the same variation in $Y \in (0, 1)$ as $F(X)$ has in the domain $X \in (0, \infty)$. The initial and boundary conditions (13–15) can be converted in the domain of Y as

$$C(Y, 0) = \frac{C_1}{C_0} + \gamma \log\frac{1}{1-Y}; \quad Y > 0 \qquad (30)$$

at

$$Y = 0, \; -(1-\delta)(1-Y)\frac{\partial C}{\partial Y} + C = f(T); \quad 0 < T \le T_0 \quad (31a)$$

$$= 0; \quad T > T_0 \quad (31b)$$

and at

$$Y = 1, (1-Y)\frac{\partial C}{\partial Y} = \text{finite}; \quad T \ge 0 \qquad (32)$$

As $Y = 1$ corresponds to $X \to \infty$, i.e. $x \to \infty$, but it is not possible to get concentration values at infinity. The values are evaluated up to some finite extent along the longitudinal direction, away from the origin with the implicit assumption that the upper boundary is placed far enough upstream for the concentration (due to the zero-order production term) to remain unchanged with time. Let the values be computed up to $x = \ell$, which corresponds to $Y = 1 - \exp(-(\ell u_0/D_0)) = Y_0$ in the domain (0, 1). Also, the analytical solutions make clear why the concentration gradient at $x \to \infty$ is considered finite, instead of, as usual, being taken as zero, in the second boundary condition (4), although condition $\partial c^*/\partial x = 0$ as $x \to \infty$ will also yield the same analytical solutions as obtained here. The finite parameter is just the zero-order production term γ in the non-dimensional form (15) or (32). Thus, for numerical computation, the condition (32) is used in the form

at

$$Y = Y_0, \quad (1 - Y_0)\frac{\partial C}{\partial Y} = \gamma \qquad (33)$$

The sizes of the intervals along T and Y-axes are chosen to satisfy the stability condition for the explicit scheme used.

Numerical example

Analytical solutions (17a) and (17b) are solved for a selected set of numerical data to illustrate the concentration distribution behaviour of source concentration at the origin satisfying mixed-type or third-type boundary condition, along sinusoidally varying groundwater velocity. Other solutions are not solved only to avoid too many illustrations and tables. The input values are : u_0 =0.01 km day^{-1}, D_0 = 0.1 km^2 day^{-1}, $\gamma_0 = 0.5 \times 10^{-5}$ km^{-3} day^{-1}, C_0 = 1.0 and C_1 = 0.1. The source of pollution is eliminated after 2000 days i.e. t_0 = 2000 days. Flow resistance coefficient m is chosen 0.0165 day^{-1}. The values of mt are chosen as 2, 5, 8, . . ., 41 and 44, for which $u(t)$ having the expression (5a) is minimum and maximum alternatively. It means the velocity has this tendency at t = 121.2, 303.0, 484.8, . . ., 2484.8 and 2666.8 days at the regular interval of 181.8 days. Let t = 121.2 days correspond to some day in the month of June during which the groundwater level is minimal and hence the velocity is also at a minimum. This period is the peak of the summer season just before the rainy season. Then, the next value t = 303.0 days corresponds to approximately the same day in the month of December, the peak of the winter season, after the rainy season, during which groundwater level and velocity are maximum. Further, the next value t = 484.8 days will correspond to almost the same date in the month of June in the next year, and so on. Supposing that the solute concentration starts spreading from the origin in the month of February (t = 0), Fig. 1a shows the concentration values in the months of June and December respectively in the second and fifth years. Figure (1b) depicts the concentration values in the same months in the seventh and eighth years, after the source of pollution at the surface has been eliminated during the month of August in the sixth year. The two figures show the nature of the mixed-type boundary condition at the origin, stated after the equations (3). In the case of the first-type boundary condition, input concentration in Fig (1a) would have been 1.0 while that in Fig. (1b) would have been zero at all the times chosen. As time passes, the region near the origin will contain fewer solute particles. In both figures, the concentration distribution is shown up to x = 10 km. The solutions (17) are compared with the results that would have been obtained in the case of initially solute free groundwater (C_1 = 0 and γ = 0).

Tables (1a) and (1b) present numerical results for the same problem in an inhomogeneous aquifer. The concentration values are obtained for the same set of data with one change; the value of γ_0 is taken as 0.2×10^{-5}, instead of as 0.5×10^{-5} (for analytical solution). This change illustrates the effect of the zero-order production term on the concentration distribution. For a smaller value of γ_0, the initial concentration will be less and so subsequent concentration values will be lower. For an initially solute free aquifer, concentration values shown by dotted curves in the Figures (1a,b) are much lower than those in the presence of a zero order production term. The numerical result for an homogeous aquifer is compared with the analytical result for the same set of data and it is found that the two coincide up to four decimal places. This excellent agreement between the two confirms the suitability of the numerical solution scheme. Table (1a) also shows that, from the origin, for example at x = 10 km, concentration values are almost the same, converging to the initial concentration at all times. This will be more evident as x > 10 km. As the numerical and analytical solutions are in good agreement, this tendency can also be seen in Fig. (1a). This justifies the implicit assumption stated for the upper boundary condition (33). Also the effect of initial groundwater velocity on the concentration distribution is also shown. Table (1b) shows that in the case of higher initial velocity, concentrations at the origin as well as at other positions decrease more rapidly as time increases i.e. a polluted aquifer will be rehabilitated sooner. For example at 2121 days at x = 4 km, the concentration is higher for u_0 = 0.05 km day^{-1} than that for u_0 = 0.01 km day^{-1}. But at t = 2666.8 days at the same position, the concentration becomes less for higher values of u_0.

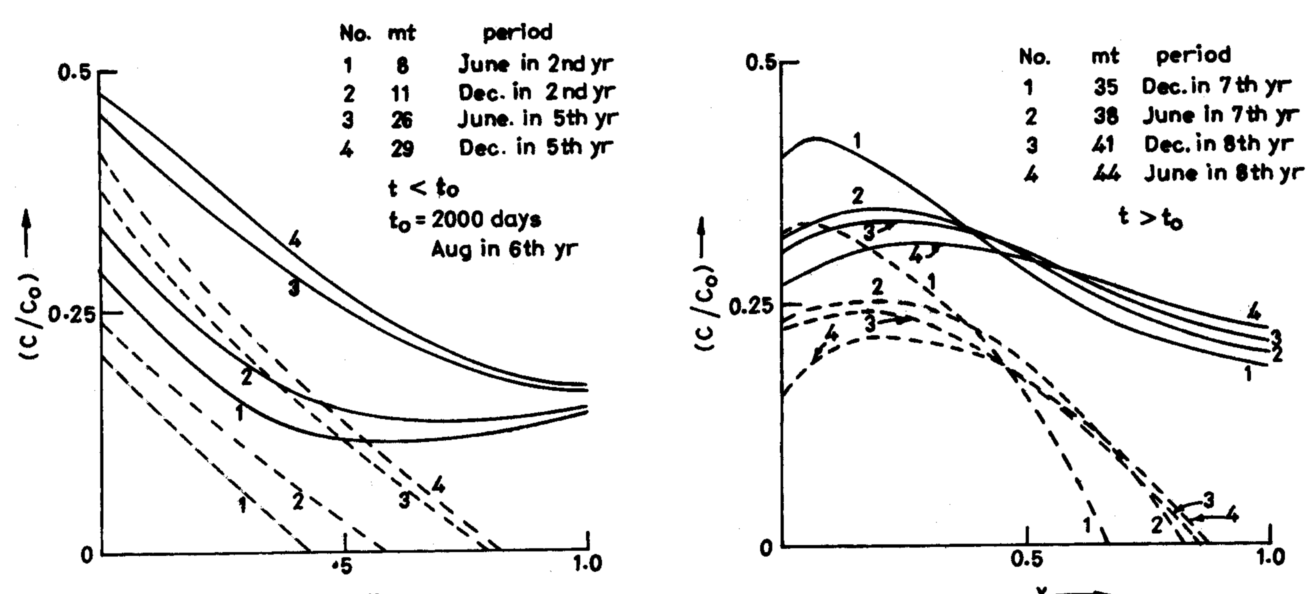

Figure 1

Table la

Concentration values, for $\gamma_0 = 0.2 \times 10^{-5}$ numerically obtained at $t \leq$ to for mixed type boundary condition at $x = 0$ along sinusoidal velocity in (i) homogeneous aquifer (a) $u_0 = 0.05$ km day^{-1}, (b) $u_0 = 0.01$ km day^{-1}, (ii) inhomogeneous aquifer for expression (26a) of F(X) and (iii) for expression (26b) of F(X)

x(km)	0.0	2	4	6	8	10
			t = 484.8 days (mt = 8)			
(i)a	.787209	.742420	.688090	.624835	.561004	.531619
(i)b	.303598	.183583	.123201	.112820	.116224	.123580
(ii)	.284812	.166481	.116892	.111981	.115886	.123181
(iii)	.307841	.184016	.121601	.112915	.116504	.123665
			t = 666.7 days (mt = 11)			
(i)a	.864646	.836130	.801437	.760870	.719742	.700825
(i)b	.337888	.218062	.141190	.116162	.116533	.124051
(ii)	.315450	.195881	.129561	.113530	.115944	.123502
(iii)	.344201	.220207	.138827	.115518	.116864	.124246
			t = 1575.8 days (mt = 26)			
(i)a	.985906	.982936	.979319	.975099	.970868	.969139
(i)b	.449478	.341042	.239297	.164552	.129612	.128844
(ii)	.414976	.302808	.206726	.145197	.122469	.125637
(iii)	.464940	.352655	.239790	.159652	.126955	.128113
			t = 1757.6 days (mt = 29)			
(i)a	.988881	.986537	.983685	.980359	.977034	.975728
(i)b	.458077	.350903	.248649	.171038	.132378	.130082
(ii)	.422632	.311446	.214396	.149856	.124058	.126208
(iii)	.474396	.363470	.249752	.165973	.129203	.128966

Table lb

Concentration values for similar cases as in Table la, but at $t > t_0$ (2000 days).

x**(km)**	**0.0**	**2**	**4**	**6**	**8**	**10**
			t = 2121.2 days (mt = 35)			
(i)a	.602168	.721449	.836177	.925144	.973003	.984814
(i)b	.378542	.378679	.281281	.195482	.144476	.136419
(ii)	.348266	.337280	.241387	.167835	.131349	.129355
(iii)	.397940	.395292	.284834	.190262	.139370	.133442
			t = 2303.0 days (mt =38)			
(i)a.	.267668	.323955	.392019	.470937	.550278	.587220
(i)b	.298726	.333716	.294562	.215741	.156619	.143945
(ii)	.276734	.303727	.255506	.183320	.139047	.133407
(iii)	.317847	.353625	.302425	.211090	.149977	.139023
			t = 2484.8 days (mt = 41)			
(i)a	.229841	.278221	.336920	.405315	.474480	.506833
(i)b	.288637	.325026	.294209	.219643	.159354	.145841
(ii)	.267662	.296548	.256165	.186495	.140888	.134466
(iii)	.307673	.345171	.303133	.215337	.152503	.140469

x(km)	0.0	2	4	6	8	10
			t = 2666.86 days (mt = 44)			
(i)a	.113062	.136879	.165864	.199792	.234310	.250647
(i)b	.252962	.291370	.286006	.233347	.173238	.155948
(ii)	.235549	.268063	.252902	.198766	.150140	.140314
(iii)	.271623	.312039	.298927	.231723	.165172	.148409

Summary and conclusions

In the present dispersion problem in a semi-infinite aquifer, deviating from the common assumption of steady groundwater velocity, two expressions for time-dependent groundwater velocity have been chosen. The expression representing sinusoidal fluctuation in velocity, at uniform time interval describes the seasonal pattern of groundwater velocity over a year in a tropical region. A direct relationship between dispersion coefficient and velocity is used. Transfomation introducing a new time variable helps in the application of Laplace transform technique to obtain analytical solutions. The semi-infinite system is subjected to (i) a zero-order production term resulting in initially solute concentrated groundwater and (ii) pulse-type input concentration at the origin. The boundary condition at the origin is both of the third-type and of the first-type. Illustrations in the case of analytical solutions and tables in the case of numerical solutions, are given only for the third-type of boundary condition at the origin. The numerical scheme is applied only after transforming the infinite-domain into a finite one. At this stage, it is important to point out that other choices for such transformation may be made, such as

$$Y = \frac{X}{1+X} \quad \text{or} \quad 1 - \frac{1}{1+X}$$

This will result in expressions for $F(Y)$ in the eqn. (29) containing exponential functions but their variation will remain the same. Although the differential equation (25) will be transformed into:

$$\frac{\partial C}{\partial T} = (1-Y)^2 \left[F(Y) \left\{ (1-Y)^2 \frac{\partial^2 C}{\partial Y^2} - (3-2Y) \frac{\partial C}{\partial Y} \right\} + \frac{d}{dY} F(Y) \left\{ (1-Y)^2 \frac{\partial C}{\partial Y} - C \right\} \right] + \gamma$$

and, similarly, initial and boundary conditions will assume forms other than those given by (30)–(32), the results will be the same.

Two expressions for the position coordinate, one increasing and the other decreasing in the domain, are chosen to show the inhomogeneity of the aquifer. In the case of an inhomogeneous aquifer, groundwater velocity and dispersion coefficient are expressed directly as either of the two expressions. In fact, similar input concentrations are introduced at different positions on the longitudinal axis along groundwater flow, due to different point sources in a particular region on the surface. The present study presents the concentration distribution behaviour of the concentration in an aquifer due to only one such input. Alternatively expressions for $V(t)$ can be chosen, if they are of practical relevance. For any such expression, the resulting equation from the transformation (9) should be solvable for the old time variable in terms of the new one so that the boundary conditions $c^*(x, t)$ are converted into $C(X, T)$. A similar condition applies to any function $f(t)$ existing in the boundary condition (3a). That is why a suitable expression for $f(t)$ has been considered when $V(t)$ decreases exponentially while $f(t) = 1$ has been taken when the form $V(t)$ is sinusoidal.

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